International Journal of Theoretical Physics, Vol. 32, No. 12, 1993

Parameter Dependence in Dynamical Models for Statevector Reduction

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Received July 22, 1993

We apply the distinction between parameter independence and outcome independence to the linear and nonlinear models of a recent nonrelativistic theory of continuous statevector reduction. We show that in the nonlinear model there is a set of realizations of the stochastic process that drives the statevector reduction for which parameter independence is violated for parallel spin components in the EPR-Bohm setup. Such a set has an appreciable probability of occurrence ($\approx 1/2$). On the other hand, the linear model exhibits only extremely small parameter dependence effects. The final section discusses the difficulties of finding a relativistic generalization of a parameter-dependent nonrelativistic theory. We identify this difficulty precisely and show how the weak parameter dependence of the linear model avoids it, provided one uses an appropriate criterion for the existence of definite outcomes.

1. NONLOCALITY, PARAMETER DEPENDENCE, AND OUTCOME DEPENDENCE

As is well known, the principal locality assumption needed to prove Bell's theorem for the stochastic case (Bell, 1971) is equivalent to the conjunction of two other assumptions, viz., in Shimony's terminology, parameter independence and outcome independence (Suppes and Zanotti, 1976; van Fraassen, 1982; Jarrett, 1984; Shimony, 1984). In view of the experimental violation of the Bell inequality, one has to give up either or both of these assumptions.

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0020-7748/93/1200-2287$07.00/0

The aim of this paper is to investigate which of these assumptions is given up by some recent models of statevector reduction. In particular, we will focus on parameter dependence. We will show that one of these models (in which the evolution of the statevector is governed by a nonlinear equation) exhibits appreciable parameter dependence effects, while the corresponding linear model does not. This is an important issue, since, as we will argue in Section 4, parameter dependence of a nonrelativistic theory prevents a genuinely relativistic generalization. [This paper abbreviates a previous paper by Ghirardi *et al.* (1993).]

To start with, let us fix our notation. We will denote by λ all parameters (which may include the quantum mechanical statevector or even reduce to it alone) that completely specify the state of an individual physical system. For simplicity we will refer to a standard EPR-Bohm setup and we will denote by

$$p_\lambda^{LR}(x, y; \mathbf{n}, \mathbf{m}) \tag{1.1}$$

the joint probability of getting the outcome x ($x = \pm 1$) in a measurement of the spin component along $\mathbf{n}$ at the left (L) and y ($y = \pm 1$) in a measurement of the spin component along $\mathbf{m}$ at the right (R) wing of the apparatus. We assume that the experimenter at L can make a free-will choice of the direction $\mathbf{n}$; and similarly for the experimenter at R and the direction $\mathbf{m}$. Both experimenters can also choose not to perform the measurement.

Bell's locality assumption can be expressed as

$$p_\lambda^{LR}(x, y; \mathbf{n}, \mathbf{m}) = p_\lambda^{L}(x; \mathbf{n}, *)p_\lambda^{R}(y; *, \mathbf{m}) \tag{1.2}$$

where the symbol * appearing on the r.h.s. denotes that the corresponding measurement is not performed. Condition (1.2) has been shown (Jarrett, 1984; Shimony, 1984) to be equivalent to the conjunction of two logically independent conditions:

$$\begin{aligned} p_\lambda^{L}(x; \mathbf{n}, \mathbf{m}) &= p_\lambda^{L}(x; \mathbf{n}, *) \\ p_\lambda^{R}(y; \mathbf{n}, \mathbf{m}) &= p_\lambda^{R}(y; *, \mathbf{m}) \end{aligned} \tag{1.3a}$$

and

$$p_\lambda^{LR}(x, y; \mathbf{n}, \mathbf{m}) = p_\lambda^{L}(x; \mathbf{n}, \mathbf{m})p_\lambda^{R}(y; \mathbf{n}, \mathbf{m}) \tag{1.3b}$$

where we have denoted, e.g., by the symbol $p_\lambda^{L}(x; \mathbf{n}, \mathbf{m})$ the probability of getting, for the given settings $\mathbf{n}, \mathbf{m}$, the outcome x at L.

Conditions (1.3a) express *parameter independence*, i.e., the requirement that the probability of getting an outcome at L (R) is independent of the setting chosen at R (L), while equation (1.3b) (*outcome independence*)

expresses the requirement that the probability of an outcome at one wing does not depend on the outcome obtained at the other wing.

This "splitting" of the locality requirement into two independent conditions is particularly useful for discussing their different conceptual implications with respect to relativity. In fact [as proved most fully by Jarrett (1984)], if conditions (1.3a) were violated, and one could control the variables λ, one could send faster-than-light signals from R (L) to L (R). On the other hand, if only condition (1.3b) is violated, then faster-than-light signaling cannot be achieved since the stochastic outcome at a wing cannot be controlled by the experimenter.

As is well known, this splitting also gives a useful contrast between standard quantum mechanics (i.e., under the assumption that the statevector $|\Psi\rangle$ is the complete state λ) and pilot wave theories, in particular that of Bohm (1952). Thus, standard quantum mechanics exhibits parameter independence [a special case of the "no-signaling theorem" (Eberhard, 1978; Ghirardi *et al.*, 1980, 1988) and outcome dependence. But for the pilot wave theory, the situation is reversed: there is outcome independence and parameter dependence (Butterfield, 1992, pp. 60–63). [Quantum statistics and, in particular, the no-signaling theorem are recovered at the phenomenological level by averaging over the controllable precise positions of the particles (Holland and Vigier, 1988, pp. 745ff).] Indeed, in any deterministic theory (i.e., one for which the range of each of the above probabilities is the set $\{0, 1\}$) there cannot be outcome dependence, so that violation of the locality requirement (1.2) implies parameter dependence.

Here we set aside "apparatus microstates"—features, μ_L and μ_R, say, of the L and R apparatuses that are not included in λ but do affect the probabilities of results. Of course some authors [especially Jarrett (1984)] allow for these. So they write probabilities $p_\lambda^{LR}(x, y; \mathbf{n}\mu_L, \mathbf{m}\mu_R)$ and then have a decomposition like that of (1.2) into (1.3a) and (1.3b), but at each value of μ_L and μ_R. Considering these probabilities raises some interesting questions (Bell, Shimony *et al.*, 1985). But following Shimony (1984), we think it is reasonable to assume we can define "surface" probabilities by averaging over the microstates, and so to consider (1.2), i.e., (1.3a) and (1.3b), as the locality condition. Accordingly, we set aside apparatus microstates in what follows.

2. A CONCISE REVIEW OF DYNAMICAL REDUCTION MODELS

Models have recently been developed which, by using stochastic modifications of Schrödinger's dynamics, imply wave packet reduction with definite pointer positions in measurement processes. More generally, they forbid the persistence of linear superpositions of macroscopically

distinguishable states, and do so without violating any established experimental facts.

The first model of this kind, quantum mechanics with spontaneous localizations (QMSL) (Ghirardi *et al.*, 1986; Bell 1987*a*), is based on the assumption that, besides the standard evolution, physical systems are subjected to spontaneous localizations occurring at random times and affecting their elementary constituents. Such processes, which we call "hittings," are formally described as follows. When the ith constituent of the system suffers a hitting the wave function changes according to

$$\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N) \to \Psi_{\mathbf{x}}(\mathbf{r}_1, \ldots, \mathbf{r}_N) = \Phi_{\mathbf{x}}(\mathbf{r}_1, \ldots, \mathbf{r}_N)/\|\Phi_{\mathbf{x}}\|$$
$$\Phi_{\mathbf{x}}(\mathbf{r}_1, \ldots, \mathbf{r}_N) = \exp[-(\alpha/2)(\mathbf{r}_i - \mathbf{x})^2]\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N) \tag{2.1}$$

Such processes occur at randomly distributed times with a mean frequency $\lambda = 10^{-16}\,\text{sec}^{-1}$. The probability density of the process occurring at point $\mathbf{x}$ is given by $\|\Phi_{\mathbf{x}}\|^2$. The localization parameter $1/\sqrt{\alpha}$ is assumed to take the value 10^{-5} cm.

The QMSL mechanism does not respect the symmetry properties of the wave function in the case of identical constituents. Its generalization, continuous spontaneous localization (CSL), does so, and has been presented and discussed in various papers (Pearle, 1989; Ghirardi *et al.*, 1990*a*; Ghirardi and Rimini, 1990). At the nonrelativistic level, there are two formulations of CSL which are equivalent from a physical point of view. However, they turn out to exhibit quite different nonlocal features. So to present our argument, we first need the details of these two formulations of CSL, which we call "the linear CSL model" and "the nonlinear CSL model," respectively.

2.1. The Linear CSL Model

The model is based on a linear stochastic evolution equation for the statevector. The evolution does not preserve the norm, but only the average value of the squared norm. The equation is

$$\frac{d|\Psi_w(t)\rangle}{dt} = \left[-\frac{i}{h}H + \sum_i A_i w_i(t) - \gamma \sum_i A_i^2\right]|\Psi_w(t)\rangle \tag{2.2}$$

In equation (2.2), the quantities A_i are commuting self-adjoint operators, while the quantities $w_i(t)$ are c-number Gaussian stochastic processes with a probability density of occurrence

$$P_{\text{Cook}}[w] = P_{\text{Raw}}[w]\,\|\,|\Psi_w(t)\rangle\|^2 \tag{2.3}$$

In equation (2.3), $P_{\text{Raw}}[w]$ is the "raw" probability density for the Gaussian process w in the interval $(0, t)$, namely (with N a normalization factor)

$$P_{\text{Raw}}[w] = \frac{1}{N} \exp\left[-\frac{1}{2\gamma} \sum_i \int_0^t d\tau \, w_i^2(\tau) \right] \tag{2.4}$$

that is, the probability density for a white noise satisfying

$$\langle\!\langle w_i(t) \rangle\!\rangle = 0, \qquad \langle\!\langle w_i(t) w_j(t') \rangle\!\rangle = \gamma \delta_{ij} \delta(t - t') \tag{2.5}$$

We assume, for the moment, that the operators A_i have a purely discrete spectrum and we denote by P_σ the projection operators on their common eigenmanifolds.

The physical meaning of the model is made precise by the following prescription: if a homogeneous ensemble (pure case) at the initial time $t = 0$ is associated with the statevector $|\Psi, 0\rangle$, then the ensemble at time t is the union of homogeneous ensembles associated with the normalized vectors $|\Psi_w(t)\rangle / \| |\Psi_w(t)\rangle \|$, where $|\Psi_w(t)\rangle$ is the solution of equation (2.2) with the assigned initial conditions and for the specific stochastic process w which has occurred in the interval $(0, t)$.

For our concerns, the relevant feature of the dynamical process (2.2) with the prescription (2.3) is that it drives the statevector of each individual member of the ensemble into one of the common eigenmanifolds of the operators A_i, with the appropriate probability. To make this clear, we consider a simplified case in which only one operator A appears in equation (2.2). The solution of this equation corresponding to the particular initial condition (involving only two eigenmanifolds of A with eigenvalues α, β)

$$|\Psi, 0\rangle = P_\alpha |\Psi, 0\rangle + P_\beta |\Psi, 0\rangle \tag{2.6}$$

when the Hamiltonian is disregarded, is

$$|\Psi_B(t)\rangle = e^{\alpha B(t) - \alpha^2 \gamma t} P_\alpha |\Psi, 0\rangle + e^{\beta B(t) - \beta^2 \gamma t} P_\beta |\Psi, 0\rangle \tag{2.7}$$

Here $B(t)$ is the Brownian process

$$B(t) = \int_0^t d\tau \, w(\tau) \tag{2.8}$$

and we have changed notation from $|\Psi_w(t)\rangle$ to $|\Psi_B(t)\rangle$ in order to stress the fact that the state at time t does not depend on the particular sample function $w(\tau)$ in the interval $(0, t)$, but only on its integral $B(t)$ of equation (2.8).

Taking into account equation (2.8) and the cooking prescription, one gets the cooked probability density for the value $B(t)$ of the Brownian

process at time t:

$$P_{\text{Cook}}[B(t)] = P_{\text{Raw}}[B(t)] \, \| |\Psi_B(t)\rangle \|^2$$

$$P_{\text{Cook}}[B(t)] = \| P_\alpha |\Psi, 0\rangle \|^2 \frac{1}{(2\pi\gamma t)^{1/2}} e^{-(1/2\gamma t)[B(t) - 2\alpha\gamma t]^2}$$

$$+ \| P_\beta |\Psi, 0\rangle \|^2 \frac{1}{(2\pi\gamma t)^{1/2}} e^{-(1/2\gamma t)[B(t) - 2\beta\gamma t]^2} \tag{2.9}$$

From (2.9) it is evident that for $t \to \infty$, the Brownian process $B(t)$ can assume only values belonging to an interval of width $(\gamma t)^{1/2}$ around either the value $2\alpha\gamma t$ or the value $2\beta\gamma t$. [Although the spread $(\gamma t)^{1/2}$ tends to ∞ for $t \to \infty$, its ratio to the distance $2(\alpha - \beta)\gamma t$ between the two peaks of the distribution tends to zero.] The corresponding probabilities are $\| P_\alpha |\Psi, 0\rangle \|^2$ and $\| P_\beta |\Psi, 0\rangle \|^2$, respectively. The occurrence of a value near to $2\alpha\gamma t$ for the random variable $B(t)$ leads, according to equation (2.7), to a statevector that, for $t \to \infty$, lies in the eigenmanifold corresponding to the eigenvalue α of A. In fact, one gets

$$\frac{\| P_\beta |\Psi_B(t)\rangle \|^2}{\| P_\alpha |\Psi_B(t)\rangle \|^2} \cong e^{-2\gamma t(\alpha - \beta)^2} \frac{\| P_\beta |\Psi, 0\rangle \|^2}{\| P_\alpha |\Psi, 0\rangle \|^2} \underset{t \to \infty}{\longrightarrow} 0 \tag{2.10}$$

Analogously, when the random variable $B(t)$ takes a value near to $2\beta\gamma t$, for $t \to \infty$, the state vector is driven into the eigenmanifold corresponding to the eigenvalue β of A.

It is then clear that the model establishes a one-to-one correspondence between the outcome (the final "preferred" eigenmanifold into which an individual statevector is driven) and the specific value (among the only ones having an appreciable proability) taken by $B(t)$ for $t \to \infty$, a correspondence irrespective of what $|\Psi, 0\rangle$ is, provided only that it has nonzero amplitudes for each eigenmanifold. In the general case of several operators A_i, a similar conclusion holds for the outcomes α_i of A_i and the corresponding Brownian processes $B_i(t)$.

This concludes the exposition of the linear CSL model. Obviously, to give a physical content to the theory one must choose the so-called preferred basis, i.e., the eigenmanifolds on which reduction takes place or, equivalently, the set of commuting operators A_i. The specific form that has been presented and shown to possess all the desired features (Pearle, 1989; Ghirardi *et al.*, 1990*a*; Ghirardi and Rimini, 1990) is obtained by identifying the discrete index i and the operators A_i of the above formulas with the continuous index $\mathbf{x}$ and the operator

$$N(\mathbf{x}) = \left(\frac{\alpha}{2\pi}\right)^{3/2} \sum_s \int d\mathbf{q} \, e^{-(\alpha/2)(\mathbf{q} - \mathbf{x})^2} a^+(\mathbf{q}, s) a(\mathbf{q}, s) \tag{2.11}$$

Here $a^+(\mathbf{q}, s)$ and $a(\mathbf{q}, s)$ are the creation and annihilation operators of a

particle at point $\mathbf{q}$ with spin component s, satisfying the canonical commutation or anticommutation relations. Correspondingly, one has a continuous family of Gaussian stochastic processes satisfying

$$\langle\!\langle w(\mathbf{x}, t)\rangle\!\rangle = 0, \qquad \langle\!\langle w(\mathbf{x}, t)w(\mathbf{y}, t')\rangle\!\rangle = \gamma\delta(\mathbf{x}-\mathbf{y})\delta(t-t') \tag{2.12}$$

The parameter α is assumed to take the same value ($10^{10}\,\mathrm{cm}^{-2}$) as in the case of QMSL, while γ is related to the frequency $\lambda = 10^{-16}\,\mathrm{sec}^{-1}$ of that model according to $\gamma = \lambda(4\pi/\alpha)^{3/2}$.

2.2. The Nonlinear CSL Model

It is possible to reformulate the theory so that the stochastic process $w(t)$ is subject to the raw probability given by equation (2.5). When one does this, one finds that the normalized state vector obeys the nonlinear stochastic equation

$$\frac{d|\Psi_w(t)\rangle}{dt} = \Bigg[-\frac{i}{h}H + \sum_i (A_i - \langle A_i\rangle)w_i(t) - \gamma\sum_i (A_i - \langle A_i\rangle)^2 + \gamma\sum_i (\langle A_i^2\rangle - \langle A_i\rangle^2)\Bigg]|\Psi_w(t)\rangle \tag{2.13}$$

where

$$\langle A_i\rangle = \langle\Psi_w(t)|A_i|\Psi_w(t)\rangle \tag{2.14}$$

It is easy to prove that the two models are equivalent in the following precise sense: for the same initial condition and for any time t, the statistical ensembles generated in the two cases are composed of subensembles, having corresponding weights equal, of systems which went through the same history in the interval $(0, t)$.

We recall that, as in the linear CSL model, for $t \to \infty$ the non-Hamiltonian part of the dynamics drives the statevector into one of the common eigenmanifolds of the operators A_i. However, for a single realization of the stochastic process $w(t)$, equation (2.13) can lead to a final statevector lying in *a different eigenmanifold*, depending on the initial condition $|\Psi, 0\rangle$.

2.3. Outcomes in Dynamical Reduction Models

Due to the choices of the parameters for QMSL and for the two models of CSL, these dynamics have the following nice features: for microscopic systems the non-Hamiltonian terms have negligible effects, while for macroscopic systems, the reduction mechanism rapidly suppresses superpositions of states in which a macroscopic number of particles are displayed by more than the characteristic localization length. In particular,

within CSL the quantity $(\alpha - \beta)^2$ which governs the damping turns out to be very large when one chooses (2.11) for the operators A_i, and therefore the suppression of the superposition occurs in a very small characteristic reduction time.

Thus it is clear how such models overcome the difficulties of quantum measurement theory. One usually assumes that different eigenstates of the measured microquantity trigger (through the system–apparatus interaction) different displacements of a macroscopic pointer from its "ready" position. These models suppress, in extremely short times and with the appropriate probability, all but one of the terms in the superposition, so that a definite outcome of the measurement emerges (See also Benatti *et al.*, 1987).

However, for the analysis in Section 3, we need to emphasize three points about these outcomes. They all follow from one main feature of the models: that exact eigenstates of the operators A_i are not obtained in finite time. Thus, consider again our simplified example within linear CSL with just two outcomes α and β which we identify with the eigenvalues of an operator A, and with the initial statevector (2.6). We stress that, once we disregard the Hamiltonian evolution, at any finite time t, none of the states evolved from this initial statevector can be exactly an eigenstate of A—although after the characteristic reduction time Δt {defined through $\exp[-2\gamma\Delta t(\alpha - \beta)^2] \ll 1$}, for all values of the Brownian process $B(\Delta t)$ which have an appreciable probability of occurrence [i.e., those for which $B(\Delta t) \approx 2\alpha\gamma\Delta t$ or $B(\Delta t) \approx 2\beta\gamma\Delta t$] the normalized statevector will have a negligible component on one of the two eigenmanifolds. Since one wants outcomes to emerge in the characteristic reduction time Δt, one must say that the "definite outcome α" has occurred when $\|P_\alpha|\Psi\rangle\|^2/\||\Psi\rangle\|^2$ is extremely close to 1: it need not exactly equal 1. (In standard quantum mechanics with the reduction postulate, one needs an analogous criterion for outcomes, since outcomes are in general related to positions of macroscopic pointers and no wave function can have compact support in configuration space for longer than an instant.)

We can now state our three points. The first is that, in principle, it could happen that even for a time larger than Δt no outcome has emerged. In fact, with the Brownian process $B(\Delta t) := (\alpha + \beta)\gamma\Delta t$, whose probability density, although very small, is not zero, one can easily show that equation (2.7) leads to a statevector which coincides, apart from a normalization factor, with the initial one. So no reduction has taken place and no outcome has been obtained. But we believe that since the probability of such a peculiar situation is extremely small, its occurrence is not a drawback of the theory.

Another peculiar situation can occur, namely the "reversal" of an outcome. Thus, suppose one has a normalized statevector $|\Psi\rangle$ which

"almost" belongs to the eigenmanifold M_α, i.e., for which $\|P_\alpha|\Psi\rangle\|^2$ is extremely close to 1, so that the outcome α has occurred. Nevertheless, there is a very small probability $\|P_\beta|\Psi\rangle\|^2$ that in the far future a value of the Brownian process $B(t)$ occurs such that in the statevector (2.7) the norm of the second term becomes much greater than the first—so that the outcome β occurs. (We stress that for an entangled statevector, such a reversal preserves the correlations implied by the statevector.)

These two points hold within all the above models of dynamical reduction. But our third point is just about linear CSL. It is that the correspondence in Section 2.1 between the outcomes and the values taken by $B(\Delta t)$ breaks down for finite times. Thus, consider the particular value $B(\Delta t) := 2\beta\gamma\Delta t$ for the Brownian process at time Δt. Such a value, when the projections on M_α and M_β of the initial state $|\Psi, 0\rangle$ have comparable norm, would yield the outcome β at this time. However, if at the initial time the two projections are so unbalanced that

$$e^{2\gamma\Delta t(\alpha-\beta)^2}[\|P_\beta|\Psi, 0\rangle\|^2/\|P_\alpha|\Psi, 0\rangle\|^2] \ll 1 \tag{2.15}$$

then at time Δt the outcome would be α. Thus the correspondence in Section 2.1 is not strictly valid for finite times. However, we stress that when (2.15) holds, the probability of occurrence at time Δt of the value $2\beta\gamma\Delta t$ for $B(\Delta t)$ is extremely small. Therefore, identification at finite times of the outcomes with the values taken by the Brownian process is legitimate to an extremely high degree of accuracy.

3. PARAMETER DEPENDENCE IN DYNAMICAL REDUCTION MODELS

In this section we will investigate the two models of CSL from the point of view of parameter dependence [Bell (1987) investigates QMSL from this point of view; his conclusions match our conclusions about the linear model in Section 3.2 below, although his mathematical apparatus is very different.] As in Section 1, we consider an EPR-Bohm setup involving two macroscopic apparatuses at L and R devised to measure $\sigma^L \cdot \mathbf{n}$ and $\sigma^R \cdot \mathbf{m}$, respectively. We will assume that the micro–macro interactions taking place at L and R that trigger reduction are governed by appropriate coupling constants g_L and g_R. So, to investigate whether there is parameter dependence, we will compare situations in which one of the coupling constants is made equal to zero (corresponding to no measurement being performed) with a situation in which it is nonzero.

For both the linear and nonlinear models, the initial situation before any measurement is completely characterized by the initial statevector $|\Psi, 0\rangle$. The later situation is determined by the evolution equation

depending on g_L and g_R, together with a realization (sample function) $w(\mathbf{x}, t)$ of the stochastic process. The various realizations have a (nonepistemic) probability of occurrence. In the nonlinear model, this is $P_{\text{Raw}}[w]$, which is independent of everything else, and, in particular, of the values taken by g_L and g_R. But in the linear model, this probability does depend on the values of g_L and g_R. This is due to the fact that the evolution of the statevector $|\Psi, 0\rangle$ depends on the coupling constants and the statevector itself enters in determining the cooked probability density of the stochastic processes. As we shall see, this difference between the two models makes for very different behavior as regards parameter dependence.

Strictly speaking, the evolution equation describing the whole process contains a Hamiltonian term describing the space propagation of the particles toward L and R, Hamiltonian terms with coupling constants g_L and g_R which describe the spin–apparatus interactions, and the reducing terms containing the stochastic process $w(\mathbf{x}, t)$. However, we will simplify our analysis: first, by disregarding the space propagation; second, by considering only those values of w that affect the reduction, i.e., the values for the spatial regions of the apparatuses and for the relevant time intervals. We will denote by w_L and w_R these restrictions of w.

To further simplify the discussion, we assume that the initial state is the singlet state and we confine our attention to the case in which both spin measurements are in the same direction, i.e., $\mathbf{n} = \mathbf{m}$. We assume that the measurement at R, if it takes place (i.e., if $g_R \neq 0$), occurs at an earlier time than the one at L.

3.1. The Case of the Nonlinear CSL Model

With these simplifications, we can easily show that the nonlinear model is parameter dependent, without solving its evolution equation explicitly. The parameter dependence follows straightforwardly from the probabilistic independence of w_L and w_R. In fact, we can simplify further by setting aside the "no outcome" and "reversing an outcome" possibilities discussed in Section 2.3. Allowing for these would not affect the argument of this subsection: it would simply mean that some of this subsection's equalities hold only to an extremely high degree of accuracy.

Assume that the initial statevector $|\Psi, 0\rangle$ is the singlet state and consider the realizations $\tilde{w}_L(\mathbf{x}, t)$ of $w_L(\mathbf{x}, t)$ that give rise to the outcome $+1$ for the left apparatus when it is triggered by $|\Psi, 0\rangle$. The probability of occurrence of such processes is 1/2. We will denote by $p^L_{|\Psi,0\rangle}(-1; g_R = 0|w_L)$ and $p^L_{|\Psi,0\rangle}(-1; g_R \neq 0|w_L)$ the conditional probability, given w_L, of the outcome -1 at left when the initial state is $|\Psi, 0\rangle$ and the R apparatus is switched off or on, respectively. We then have

$$p^{\mathrm{L}}_{|\Psi,0\rangle}(-1; g_{\mathrm{R}} = 0|\tilde{w}_{\mathrm{L}}) = 0 \tag{3.1}$$

We now evaluate the probability $p^{\mathrm{L}}_{|\Psi,0\rangle}(-1; g_{\mathrm{R}} \neq 0|\tilde{w}_{\mathrm{L}})$. Since $g_{\mathrm{R}} \neq 0$ and the measurement at R occurs before the one at L, we have to take into account the possible realizations of the stochastic process at R. Let us consider the realizations $\tilde{w}_{\mathrm{R}}(\mathbf{x}, t)$ of $w_{\mathrm{R}}(\mathbf{x}, t)$ that, when triggered by the singlet state, yield the outcome $+1$ at R. When one of these processes $\tilde{w}_{\mathrm{R}}$ occurs, the outcome at L turns out to be -1 irrespective of the particular realization of the stochastic process w_{L} and therefore also for all processes $\tilde{w}_{\mathrm{L}}$ considered above. To understand this, recall the point at the end of Section 2.2: that within the nonlinear model, the same stochastic process at L can give rise to different outcomes, depending on the statevector which triggers the apparatus at L. As a consequence one has

$$p^{\mathrm{L}}_{|\Psi,0\rangle}(-1; g_{\mathrm{R}} \neq 0|\tilde{w}_{\mathrm{L}} \;\&\; \tilde{w}_{\mathrm{R}}) = 1 \tag{3.2}$$

Since the probability of occurrence of a process $\tilde{w}_{\mathrm{R}}$ is equal to $1/2$ and is independent of the particular realization $\tilde{w}_{\mathrm{L}}$, and since if w_{R} is not one of the $\tilde{w}_{\mathrm{R}}$, then outcome -1 on the left cannot occur (barring improbable exceptions), one has

$$p^{\mathrm{L}}_{|\Psi,0\rangle}(-1; g_{\mathrm{R}} \neq 0|\tilde{w}_{\mathrm{L}}) = 1/2 \tag{3.3}$$

We stress that the difference of the probabilities is appreciable,

$$0 = p^{\mathrm{L}}_{|\Psi,0\rangle}(-1; g_{\mathrm{R}} = 0|\tilde{w}_{\mathrm{L}}) \neq p^{\mathrm{L}}_{|\Psi,0\rangle}(-1; g_{\mathrm{R}} \neq 0|\tilde{w}_{\mathrm{L}}) = 1/2 \tag{3.4}$$

and that the probability of occurrence of these realizations $\tilde{w}_{\mathrm{L}}$ is also appreciable ($=1/2$). Thus the nonlinear CSL model exhibits parameter dependence.

3.2. The Case of the Linear CSL Model

For the linear model, we can easily solve the evolution equation, and thereby show parameter independence in the $t \to \infty$ limit, once we simplify the description by considering only the spin Hilbert space. This simplification will not affect the issue of parameter independence, provided one correspondingly changes the value of the parameter γ in such a way that reduction takes place within times characteristic for a macroscopic apparatus.

Thus one has, in the case in which both apparatuses are switched on ($g_{\mathrm{R}} \neq 0$ and $g_{\mathrm{L}} \neq 0$), a linear dynamical equation analogous to (2.2):

$$\frac{d|\Psi_{w_{\mathrm{L}},w_{\mathrm{R}}}(t)\rangle}{dt} = \{[(\sigma^{\mathrm{L}} \cdot \mathbf{n})w_{\mathrm{L}}(t) - \gamma] + [(\sigma^{\mathrm{R}} \cdot \mathbf{m})w_{\mathrm{R}}(t) - \gamma]\}|\Psi_{w_{\mathrm{L}},w_{\mathrm{R}}}(t)\rangle \tag{3.5}$$

with

$$\langle\!\langle w_{\mathrm{L}}(t)\rangle\!\rangle = 0;\ \langle\!\langle w_{\mathrm{R}}(t)\rangle\!\rangle = 0;\ \langle\!\langle w_{\mathrm{L}}(t)w_{\mathrm{R}}(t')\rangle\!\rangle = \gamma\delta_{\mathrm{L,R}}\delta(t-t') \tag{3.6}$$

The probability distribution of the stochastic processes is obtained through the cooking procedure. To compare this case with the one in which $g_{\mathrm{R}} = 0$, one has to consider another stochastic equation, i.e.,

$$\frac{d|\Psi_{w_{\mathrm{L}}}(t)\rangle}{dt} = \{(\sigma^{\mathrm{L}}\cdot\mathbf{n})w_{\mathrm{L}}(t) - \gamma\}|\Psi_{w_{\mathrm{L}}}(t)\rangle \tag{3.7}$$

The solutions of equations (3.5) and (3.7) at time t for the same initial conditions are

$$|\Psi_{B_{\mathrm{L}},B_{\mathrm{R}}}(t)\rangle = \exp[F_{\mathrm{L}B_{\mathrm{L}}}(t)]\exp[F_{\mathrm{R}B_{\mathrm{R}}}(t)]\ |\Psi,0\rangle \tag{3.8}$$

and

$$|\Psi_{B_{\mathrm{L}}}(t)\rangle = \exp[F_{\mathrm{L}B_{\mathrm{L}}}(t)]|\Psi,0\rangle \tag{3.9}$$

respectively. [In equation (3.8) and following, we change notation for the same reason as we did in equation (2.7).] In equations (3.8) and (3.9) we have put

$$F_{\mathrm{L}B_{\mathrm{L}}}(t) = \sigma^{\mathrm{L}}\cdot\mathbf{n}B_{\mathrm{L}}(t) - \gamma t; \qquad F_{\mathrm{R}B_{\mathrm{R}}}(t) = \sigma^{\mathrm{R}}\cdot\mathbf{n}B_{\mathrm{R}}(t) - \gamma t \tag{3.10}$$

where

$$B_{\mathrm{L}}(t) = \int_0^t d\tau\ w_{\mathrm{L}}(\tau), \qquad B_{\mathrm{R}}(t) = \int_0^t d\tau\ w_{\mathrm{R}}(\tau) \tag{3.11}$$

We come back now to equation (3.5) and we evaluate the cooked probability density of occurrence of the Brownian processes $B_{\mathrm{L}}(t)$ and $B_{\mathrm{R}}(t)$ by multiplying the raw probability density by the square of the norm of the statevector (3.8). As usual we have

$$P_{\mathrm{Cook}}[B_{\mathrm{L}}(t)\ \&\ B_{\mathrm{R}}(t)] = P_{\mathrm{Raw}}[B_{\mathrm{L}}(t)\ \&\ B_{\mathrm{R}}(t)]\,\big\|\,|\Psi_{B_{\mathrm{L}},B_{\mathrm{R}}}(t)\rangle\big\|^2 \tag{3.12}$$

and

$$P_{\mathrm{Raw}}[B_{\mathrm{L}}(t)\ \&\ B_{\mathrm{R}}(t)] = P_{\mathrm{Raw}}[B_{\mathrm{L}}(t)]P_{\mathrm{Raw}}[B_{\mathrm{R}}(t)] \tag{3.13}$$

Taking into account equation (3.8), one then gets from (3.12)

$$\begin{aligned} &P_{\mathrm{Cook}}[B_{\mathrm{L}}(t)\ \&\ B_{\mathrm{R}}(t)] \\ &\quad = P_{\mathrm{Raw}}[B_{\mathrm{L}}(t)]P_{\mathrm{Raw}}[B_{\mathrm{R}}(t)]\,\big\|\,|\Psi_{B_{\mathrm{L}},B_{\mathrm{R}}}(t)\rangle\big\|^2 \\ &\quad = P_{\mathrm{Raw}}[B_{\mathrm{L}}(t)]\,\big\|\exp[F_{\mathrm{L}B_{\mathrm{L}}}(t)]|\Psi,0\rangle\big\|^2 \\ &\qquad \times P_{\mathrm{Raw}}[B_{\mathrm{R}}(t)]\left\|\frac{\exp[F_{\mathrm{R}B_{\mathrm{R}}}(t)]\exp[F_{\mathrm{L}B_{\mathrm{L}}}(t)]|\Psi,0\rangle}{\big\|\exp[F_{\mathrm{L}B_{\mathrm{L}}}(t)]|\Psi,0\rangle\big\|}\right\|^2 \end{aligned} \tag{3.14}$$

Let us consider the marginal cooked probability density of $B_L(t)$:

$$
\begin{aligned}
&P^{\#}_{\text{Cook}}[B_L(t)] \\
&\quad = \int d[B_R(t)]\, P_{\text{Cook}}[B_L(t)\ \&\ B_R(t)] \\
&\quad = P_{\text{Raw}}[B_L(t)]\, \|\exp[F_{LB_L}(t)]|\Psi, 0\rangle\|^2 \\
&\qquad \times \int d[B_R(t)]\, P_{\text{Raw}}[B_R(t)] \left\| \frac{\exp[F_{RB_R}(t)]\, \exp[F_{LB_L}(t)]|\Psi, 0\rangle}{\|\exp[F_{LB_L}(t)]\, |\Psi, 0\rangle\|} \right\|^2
\end{aligned}
\tag{3.15}
$$

Since the equation

$$
\frac{d|\Psi_{w_R}(t)\rangle}{dt} = \{(\sigma^R \cdot \mathbf{n}) w_R(t) - \gamma\}|\Psi_{w_R}(t)\rangle \tag{3.16}
$$

preserves the stochastic average of the square of the norm of the statevector, the last integral in equation (3.15) takes the value 1. This means that $P^{\#}_{\text{Cook}}[B_L(t)]$ turns out to equal $P_{\text{Cook}}[B_L(t); *]$, i.e., the cooked probability density of occurrence of the Brownian process $B_L(t)$ for the same initial condition if the process were described by equation (3.7), i.e., if the apparatus at R were switched off.

But now recall from Section 2.1 that within linear CSL there is a one-to-one correspondence between the outcome at left (right) at $t = \infty$ and the specific value taken by the Brownian process $B_L(t)$ [$B_R(t)$] for $t \to \infty$. So the above proof that $P^{\#}_{\text{Cook}}[B_L(t)]$ equals $P_{\text{Cook}}[B_L(t); *]$ amounts to a proof that linear CSL exhibits parameter independence at the $t = \infty$ limit.

As we discussed in Section 2.3, when one considers a *finite* time t of the order of or greater than the characteristic reduction time Δt, the situation is more complicated: the one-to-one correspondence between the outcomes and the values taken by the Brownian process is only approximate (though valid to an extremely high degree of accuracy). As a consequence, linear CSL does not enjoy strict parameter independence at finite times. To clarify this point, consider the values $B_L(t) = 2\gamma t$ and $B_R(t) = 4\gamma t$ for the Brownian processes at time t. The cooked probability density of occurrence of such values at the *finite* time t, though extremely small, is not exactly zero. Going through arguments similar to those of Section 2.3, one could show that these values lead, through equation (3.8), to a statevector at t which corresponds to the outcomes $+1$ at right and -1 at left, respectively. On the other hand, for the case in which $g_R = 0$, the substitution of $B_L(t) = 2\gamma t$ in equation (3.9) leads, at time t, to a statevector corresponding to the outcome $+1$ at left. Thus, there are values of the Brownian process $B_L(t)$ for which the outcome at left depends on

whether g_R is equal to zero or not. So there is parameter dependence at the level of individual $B(t)$'s. However, given $B_L(t)$, this happens only for values $B_R(t)$ of the Brownian process at right such that the cooked conditional probability $P_{Cook}[B_R(t)|B_L(t)]$ is extremely small. This in turn implies that the model exhibits only negligibly small parameter dependence effects.

To conclude, although the linear CSL model exhibits parameter dependence at finite times, these effects are extremely small compared with what happens for the nonlinear CSL model. In the next section, we discuss how this difference affects the prospects of a relativistic generalization.

4. PARAMETER DEPENDENCE AND RELATIVISTIC INVARIANCE REQUIREMENTS

First of all, we must stress that in both the linear and nonlinear models (and indeed in QMSL), one cannot take advantage of the parameter dependence for superluminal communication—simply because each of these models recovers the quantum mechanical no-signaling theorem (see end of Section 1) at the ensemble level by averaging over the realizations of its stochastic processes.

However, if a nonrelativistic theory is parameter dependent, there are grave difficulties in giving what we will call a genuinely relativistic generalization. Here we understand "genuinely relativistic" rather strongly: it excludes theories (like Lorentz's classical electromagnetic theory, or the quantum field theory version of the de Broglie–Bohm pilot wave theory) that have a preferred frame which cannot be discovered by experiments. We favor genuinely relativistic theories. [We recall that Bell, despite his deep appreciation of the de Broglie–Bohm theory, believed that to make the Lorentz group phenomenological in this way "is an incredible position to take—I think it is quite logically consistent, but when one sees the power of the hypothesis of Lorentz invariance in modern physics, I think you just can't believe in it" (Bell, 1989, p. 13).] So we will first give a general description of these difficulties, and then consider the nonlinear and linear models.

We consider a theory formulated in a Galilean context in which there is an absolute time order and in which future events cannot influence past events. As a consequence, even though the theory exhibits parameter dependence, the setting of the parameters can only influence future events. We again consider an EPR-Bohm setup with measurements of spin components and we assume that the R measurement takes place before the L measurement, i.e., $t_R < t_L$. Starting from a given reference frame O, we assume that there are values of the parameter λ that have a nonzero

probability of occurrence and are such that

$$p^{\mathrm{L}}_{\lambda}(x; \mathbf{n}, \mathbf{m}) \neq p^{\mathrm{L}}_{\lambda}(x; \mathbf{n}, *) \tag{4.1a}$$

while

$$p^{\mathrm{R}}_{\lambda}(y; \mathbf{n}, \mathbf{m}) = p^{\mathrm{R}}_{\lambda}(y; *, \mathbf{m}) \tag{4.1b}$$

Of course, since the theory is Galilean, we cannot discuss full Lorentz invariance for it. But there is a residue, or an analog of Lorentz invariance which can be considered. Thus if the two regions R and L are very far apart from each other, there are reference frames (say, O') moving even very slowly with respect to O, for which the temporal order of the measurement events (R, t_{R}) and (L, t_{L}) is reversed, i.e., $t'_{\mathrm{R}} > t'_{\mathrm{L}}$. Since the probability of occurrence of an event is an objective fact (i.e., it cannot depend on the reference frame considered), it must hold also for O' that

$$p'^{\mathrm{L}}_{\lambda'}(x'; \mathbf{n}', \mathbf{m}') \neq p'^{\mathrm{L}}_{\lambda'}(x'; \mathbf{n}', *) \tag{4.2}$$

where λ' is the parameter which identifies for O' the systems which are identified by λ for O, and the primes on p, x, $\mathbf{n}$, and $\mathbf{m}$ have an obvious meaning. [So (4.2) merely says that both observers can look at the same systems and agree on the objective probabilities.] However, as already remarked, for O', one has $t'_{\mathrm{R}} > t'_{\mathrm{L}}$. If the theory were invariant for the O–O' transformation, an analogous situation would occur also for the observer O for some values of λ. But this contradicts assumption (4.1a) expressing that for O the past cannot have a parametric dependence on the future. So in this sense, there cannot be a genuinely relativistic generalization of a parameter-dependent theory.

This argument is readily applied to the nonlinear CSL model. Using again the notation of Section 3.1, we look at the experiment from a reference frame in slow motion with respect to O, for which the event (L, t_{L}) precedes (R, t_{R}). Consistency of the probabilities of the outcomes implies that

$$p'^{\mathrm{L}}_{|\Psi',0\rangle}(-1; g_{\mathrm{R}} \neq 0 | \tilde{w}'_{\mathrm{L}}) \neq p'^{\mathrm{L}}_{|\Psi',0\rangle}(-1; g_{\mathrm{R}} = 0 | \tilde{w}'_{\mathrm{L}}) \tag{4.3}$$

where $\tilde{w}'_{\mathrm{L}}(\mathbf{x}', t') = \tilde{w}_{\mathrm{L}}(\Lambda^{-1}(\mathbf{x}', t'))$ even though, according to O', the measurement at right occurs later than the measurement at left. So if the model was invariant for the O–O' transformation, there would be instances in which the same situation would occur for observer O—so that for O an event could influence the outcome of a process which precedes it, contradicting the nonlinear model's absolute time order. Thus we conclude that the nonlinear CSL model is not a good starting point for building up a genuinely relativistic model of dynamical reduction. In fact, as discussed elsewhere (Ghirardi *et al.*, 1990*c*), any such attempt meets difficulties (such

as the dynamical equation being nonintegrable) which are the technical counterpart of the argument above.

So what about linear CSL? First of all, we stress that there is a genuinely relativistic generalization [which we call relativistic linear CSL (Pearle, 1990; Ghirardi *et al.*, 1990*b*)] of this model, in spite of the fact (Section 3.2) that it exhibits (extremely small) parameter-dependent effects. To clarify how this can happen, we first need to recall a criterion for the existence of elements of physical reality in a system that is appropriate in a relativistic context.

Within the scheme of quantum field theory let A be a local observable (corresponding to a Lorentz scalar) with a discrete eigenvalue α and P_α at projection operator on the corresponding eigenmanifold M_α. As is well known, the assumption that wave packet reduction occurs in each reference frame along $t = \text{const}$ hyperplanes (which is an invariance requirement) implies that the value $\|P_\alpha|\Psi\rangle\|^2$ varies appreciably between relatively moving observers (Bloch, 1967; Hellwig and Kraus, 1970; Aharonov and Albert, 1980). So there is a threat that different observers disagree about the objective matter of whether there is an "element of physical reality" corresponding to the value α of the local observable A. However, a criterion that overcomes this threat has been introduced in work on relativistic linear CSL. According to this criterion, an element of physical reality corresponding to the value α of the local observable A exists only when $\|P_\alpha|\Psi\rangle\|^2$ is extremely close to 1 *for all observers*. As shown in (Ghirardi *et al.*, 1990*b*,*c*), the dynamics of relativistic linear CSL, taken together with this criterion, implies that macroscopic objects almost always have definite macroscopic properties.

Let us now apply this criterion to a case of parameter dependence in our EPR-Bohm setup. As before, let $\tilde{w}_L$ & $\tilde{w}_R$ be such that the L outcome is $+1$ or -1 according as g_R is zero or not; for O, the R measurement (R, t_R) happens before the L measurement (L, t_L), while for O', (L, t_L) precedes (R, t_R); and we consider a case in which g_R is nonzero. According to our criterion, O and O' *agree* in saying that no definite outcome has been obtained at left, since $\|P_{+L}|\Psi\rangle\|^2 \approx 0$, while $\|P'_{+L}|\Psi'\rangle\|^2 \approx 1$. This peculiar situation, that a measurement gives no definite outcome, is very improbable, since linear CSL exhibits only extremely small parameter-dependent effects (Section 3.2). And of course, nonrelativistic dynamical reduction models also allow this situation, again with very low probability (this was the first of Section 2.3's three points). Accordingly, we believe that allowing this situation is not a drawback of relativistic linear CSL.

To conclude: we have argued that the extremely weak parameter dependence of the linear model enables it to have a genuinely relativistic

generalization, once we use an appropriate criterion for the existence of definite outcomes.

ACKNOWLEDGMENTS

This work was supported in part by the Trieste Section of the I.N.F.N. Two of us (J.B. and G.N.F.) acknowledge financial support from the Interdisciplinary Laboratory for Natural and Humanistic Sciences of the ISAS of Trieste which made possible the collaboration which led to the present work. Two of us (G.C.G. and J.B.) thank the IQSA Organizing Committee for such generous and enjoyable conference arrangements. J.B. thanks the British Academy, Leverhulme Trust and Mrs. L. D. Rope Third Charitable Settlement for supporting sabbatical leave.

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